Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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Centre Number | Candidate Number

# Pearson Edexcel International GCSE

**Time** 2 hours

**Paper reference** **4PM1/02**

# Further Pure Mathematics

## PAPER 2

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

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## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** $= 4\pi r^2$

**Curved surface area of cone** $= \pi r \times$ slant height

**Volume of sphere** $= \frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

### Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** Find the set of values of $k$ for which the equation

$$2kx^2 + 5kx + 5k - 3 = 0 \qquad \text{where } k \neq 0$$

has real roots.

**(4)**

**(Total for Question 1 is 4 marks)**

**2** A particle $P$ moves along the $x$-axis. At time $t$ seconds, the displacement, $x$ metres, of $P$ from the origin $O$ is given by

$$x = t^4 - 13.5t + 12$$

(a) Find the velocity, in m/s, of $P$ when $t = 3$

**(2)**

(b) Find the value of $t$ for which $P$ is instantaneously at rest.

**(2)**

(c) Find the acceleration, in m/s$^2$, of $P$ when $t = 2$

**(2)**

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**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3** $O$, $A$ and $B$ are fixed points such that

$$\overrightarrow{OA} = (p\mathbf{i} - 4\mathbf{j}) \qquad \overrightarrow{OB} = \mathbf{i} + (2p + 1)\mathbf{j}$$

Given that $\sqrt{2}\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right|$ and $p > 0$

(a) find the value of $p$

**(4)**

Using this value of $p$

(b) find a unit vector that is parallel to $\overrightarrow{AB}$

**(5)**

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4**

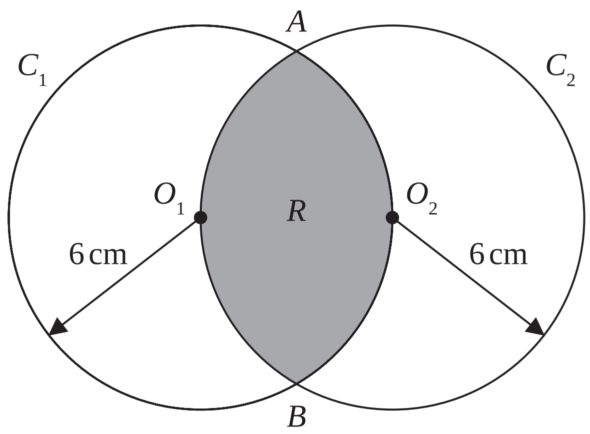


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**Figure 1**

Figure 1 shows two circles, $C_1$ and $C_2$, each with a radius of 6 cm.

The centre of $C_1$ is $O_1$ such that $O_1$ lies on $C_2$

The centre of $C_2$ is $O_2$ such that $O_2$ lies on $C_1$

The circles intersect at the points $A$ and $B$ and enclose the region $R$, shown shaded in Figure 1

The area of region $R$ is $P\,\text{cm}^2$

Find the exact value of $P$, giving your answer in the form $a\pi - b\sqrt{c}$ where $a$, $b$ and $c$ are integers.

**(7)**

**Question 4 continued**

**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 7 marks)**

**5** The roots of the quadratic equation $2x^2 + (6 + 2p)x + 2p = 0$ are $\alpha$ and $\beta$

(a) Write down an expression in terms of $p$ for

(i) $\alpha + \beta$ (ii) $\alpha\beta$

**(2)**

(b) Show that $(\alpha - \beta)^2 = 9 + 2p + p^2$

**(4)**

Given that $(\alpha - \beta) = 3$

(c) find the possible values of $p$

**(3)**

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**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 9 marks)**

**6** (a) Using a formula from page 2, show that $\cos 2A = 1 - 2\sin^2 A$

**(2)**

The finite region $R$ is bounded by the curve with equation $y = 3 + 2\sin x$, the $x$-axis,

the $y$-axis and the line with equation $x = \dfrac{\pi}{4}$

The region $R$ is rotated through 360° about the $x$-axis.

(b) Use calculus to find the volume of the solid generated.
Give your answer to the nearest integer.

**(6)**

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**Question 6 continued**

**(Total for Question 6 is 8 marks)**

**7** (i) (a) Using a formula from page 2, show that

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

**(2)**

Given that $\tan 2\alpha = 1$

(b) show that $\tan\alpha = a \pm \sqrt{b}$ where $a$ and $b$ are integers whose values need to be found.

**(3)**

(ii) (a) Using formulae from page 2, show that $\cos(x - 30)^\circ = \sin(x + 30)^\circ$ can be written as $\tan x^\circ = 1$

**(4)**

(b) Hence, or otherwise, solve

$$\cos(2y - 30)^\circ = \sin(2y + 30)^\circ \qquad \text{for } -90 < y \leqslant 90$$

**(2)**

**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 11 marks)**



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**8**

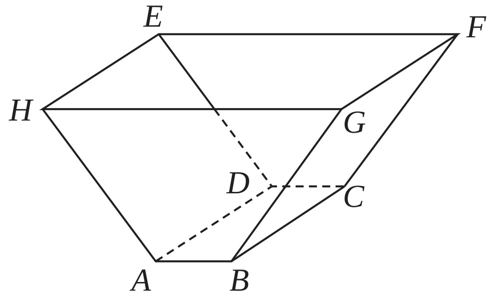


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**Figure 2**

Figure 2 shows a waste paper basket in the shape of a right prism with 5 faces and a cross section that is a trapezium. The top, *EFGH*, of the waste paper basket is open.

The base of the prism *ABCD* is a rectangle with

$$AB = DC = 2x\,\text{cm} \quad \text{and} \quad AD = BC = h\,\text{cm}$$

The cross sections *HGBA* and *EFCD* are such that

$$EF = HG = 8x\,\text{cm} \quad \text{and} \quad AH = BG = CF = DE = 5x\,\text{cm}$$

The top, *EFGH*, of the waste paper basket is such that

$$EH = FG = h\,\text{cm}$$

The volume of the waste paper basket is $2250\,\text{cm}^3$

The total surface area of the 5 faces of the waste paper basket is $S\,\text{cm}^2$

(a) Show that $S = 40x^2 + \dfrac{1350}{x}$

**(5)**

Given that $x$ can vary,

(b) use calculus, to find, to 3 significant figures, the value of $x$ for which $S$ is a minimum.
Justify that this value of $x$ gives a minimum value of $S$

**(5)**

(c) Find, to 3 significant figures, the minimum value of $S$

**(2)**

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**Question 8 continued**

**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 12 marks)**

**9** The straight line $L_1$ passes through the point $A$ with coordinates (4, 7) and has gradient $m$, where $m < 0$

Another straight line $L_2$ is perpendicular to $L_1$ and passes through the point $B$ with coordinates $(4, k)$ where $k \neq 7$

The lines $L_1$ and $L_2$ intersect at the point $C$.

Given that the $y$ coordinate of $C$ is $Y$

(a) show that $Y = \dfrac{7 + m^2k}{m^2 + 1}$

**(7)**

Given that the triangle $ABC$ is isosceles,

(b) find the value of $m$

**(5)**

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**Question 9 continued**

**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 12 marks)**

**10** Solve the equation

$$\log_4 x + \log_{16} x + \log_2 x = 10.5$$

Show your working clearly.

**(5)**

**Question 10 continued**

**(Total for Question 10 is 5 marks)**

**11** A curve $C$ has equation

$$y = \frac{(2a-1)x+1}{ax-6} \qquad \text{where } a \text{ is a constant and } x \neq \frac{6}{a}$$

(a) Find $\dfrac{dy}{dx}$

**(3)**

The curve crosses the $y$-axis at the point $A$.

The normal to $C$ at the point $A$ is the line $l$ with equation $66y - 72x + 11 = 0$

Show that

(b) (i) $a = 3$

**(4)**

(ii) the equation of $C$ is $y = \dfrac{5x+1}{3x-6}$ where $x \neq 2$

**(1)**

(c) Using the axes on the opposite page, sketch $C$, showing clearly the asymptotes with their equations and the coordinates of the points where $C$ crosses the coordinate axes.

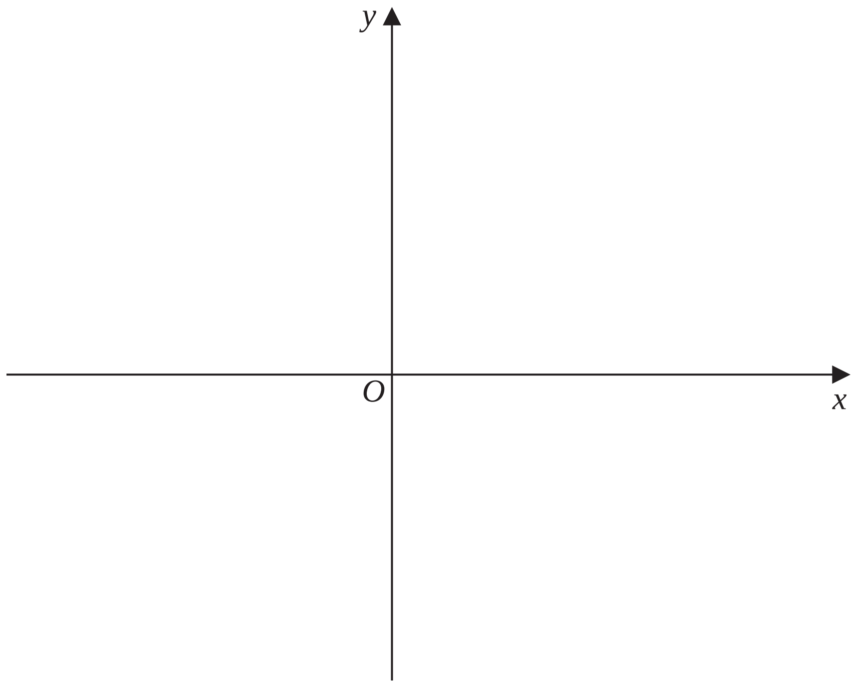
**(5)**

The line $l$ meets $C$ again at the point $D$.

(d) Find the $x$ coordinate of $D$.
Give your answer as an improper fraction.

**(4)**

**Question 11 continued**



**Question 11 continued**





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**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 17 marks)**

**TOTAL FOR PAPER IS 100 MARKS**